ANALYSIS OF A THREE-DIMENSIONAL ARBITRARILY-SHAPED DIELECTRIC OR BIOLOGICAL BODY INSIDE A RECTANGULAR WAVEGUIDE

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ABSTRACT

This paper presents a method for the analysis of three-dimensional arbitrarily-shaped dielectric obstacles inside a rectangular waveguide. The moment method is used to solve an integral equation formulated with a dyadic Green's function.

Introduction

Waveguide obstacles and discontinuities, including the dielectric type to be discussed in this paper, are long standing problems in electromagnetic theory. Although many two-dimensional problems have been solved, the general three-dimensional problems remain unsolved[1]. This paper presents a successful use of the dyadic Green's function in the analysis of a three-dimensional arbitrarily-shaped dielectric or biological body inside a rectangular waveguide. Major difficulties include a correct expression for the Green's function and the treatment of the extremely slow convergence of the double infinite series in the formulation.

The Integral Equation and the Dyadic Green's Function

The problem to be considered is shown in Figure 1. The dielectric body is in general heterogeneous. The volume equivalence principle can be shown to be valid in the bounded as well as the unbounded space. An integral equation can be formulated as follows:

$$j\omega\mu_o \int_V \underline{\underline{G}}(\underline{r},\underline{r}') \cdot \underline{J}(\underline{r})\, dV' + \frac{\underline{J}(\underline{r})}{j\omega[\varepsilon(\underline{r}) - \varepsilon_1]} + \frac{J_z(\underline{r})\hat{z}}{j\omega\varepsilon(\underline{r})} = -\underline{E}^i(\underline{r}), \tag{1}$$

where $e^{-j\omega t}$ convention is used, and the equivalent current $\underline{J}$ is

$$\underline{J}(\underline{r}) = -j\omega\,[\varepsilon\,(\underline{r}) - \varepsilon_1]\,\underline{E}(\underline{r})\ . \tag{2}$$

The dyadic Green's function $\underline{\underline{G}}$ had been presented and later corrected by Tai[2] in a short form. The explicit expression, which has never been correctly exhibited in the literature, is a double infinite series of 9 dyadic components summing up all the waveguide modal contributions.

Solution By The Moment Method

Equation (1) is an integral equation which can be solved by the standard method of moments. In the present analysis, the dielectric body is divided into rectangular-sided cells of constant dimensions. The equivalent current $\underline{J}$ in Equation (2) can be expanded in terms of a set of basis functions which are pulse functions.

We can generate a set of linear equations by performing weighted scalar product on Equation (1). The unknown current or electric field intensity can then be solved on a digital computer.

A major difficulty was encountered in the process of generating the matrix elements. The dyadic Green's function is in the form of a double infinite series of slow convergence. It was observed that the convergence is slower for obstacles of higher dielectric constant and is extremely slow for the self cells or diagonal elements in the matrix. Figure 2 shows a case in which a diagonal matrix element is not convergent even after 140 x 140 terms have been included in the calculations. The difficulty was overcome with a partial summation technique which transformed the expression into either a closed form or a single infinite series. It can be seen in Figure 2 that convergence is achieved with about 20 x 20 terms when the partial summations technique is employed.

Numerical Examples and Supporting Measurements

Three cases, as shown in Figure 3, are presented. All of the test cases consist of homogeneous dielectric bodies with rectangular-sided walls aligned with the waveguide walls. This choice of geometry conformal to the Cartesian coordinate is mainly for the sake of simplicity in data management and should not result in any significant loss of generality as was noted in the free space case. It was observed that the linear cell dimensions should be $\lambda/2$ (λ being the wavelength inside the dielectric body) or less in order to yield accurate data. Figure 4 shows good agreement in the reflection and transmission properties of Case B between a 12-cell calculation and the measured data using a model made of a silica compound. A 12-cell calculation from Case A, being also a case of low dielectric constant, yields a power reflection coefficient of 0.114 at 2.65 GHz, dropping down to 0.035 at 3.5 GHz, which was verified experimentally with a paraffin wax model.

Case C involves a model of high and frequency-dependent constant for which more cells are needed for calculation. The experimental model was made by mixing water, powdered polyethylene and a modeling compound called "Super Stuff". The measured and calculated data are compared in Figure 5, in which the agreement is not as good as the low dielectric-constant cases. This degraded accuracy of the calculation was primarily due to the large cell dimension in terms of the wavelength inside the dielectric body. For example, the y dimension of each cell is 0.8833 cm, which is about 0.52λ at 2.5 GHz and 0.68λ at 3.1 GHz.

Satisfactory convergence of the present numerical model as a function of the number of cells used has been observed in these cases. In addition to the rapid convergence of reflection and transmission coefficient, the field distribution also converges fairly well. Figure 6 shows two typical examples for medium and high dielectric constants. Measurements of the temperature profile on the surface of the dielectric body were also conducted for Case C at several frequencies using thermographic paper. The calculated distribution of dissipated power agrees reasonably well with the heating pattern recorded on the thermographic paper.

Concluding Remarks

A general three-dimensional waveguide dielectric obstacle has been successfuly treated. This general method can be applied to a number of waveguide problems including the waveguide enzyme-inactivation problem for which this study was initiated. An immediate extension of this technique would be to ferromagnetic obstacles.

References

1. P. Silvester and Z.J. Csendes, "Numerical Modeling of Passive Microwave Devices," IEEE Trans. Microwave Theory and Tech. MTT-22, pp 190-201, March 1974.
2. C.T. Tai, "On the Eigen-Function Expansions of Dyadic Green's Functions," Proc. of IEEE, vol. 61, p. 480, April 1973.

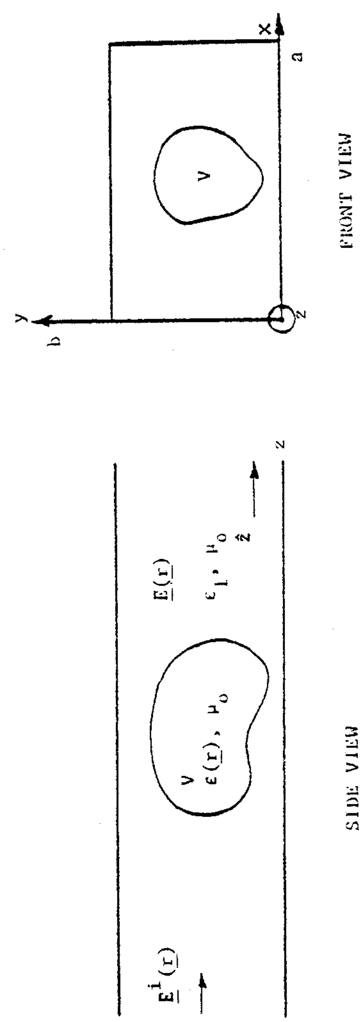


Figure 1. A three-dimensional arbitrarily-shaped heterogeneous dielectric body illuminated inside a rectangular waveguide.

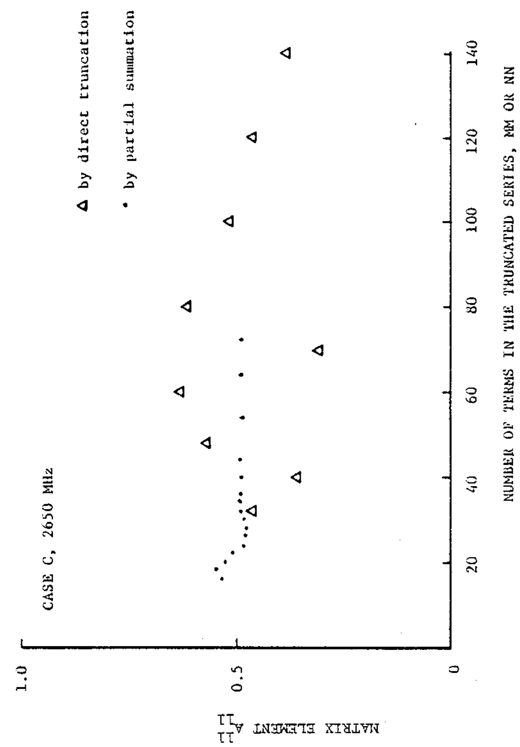
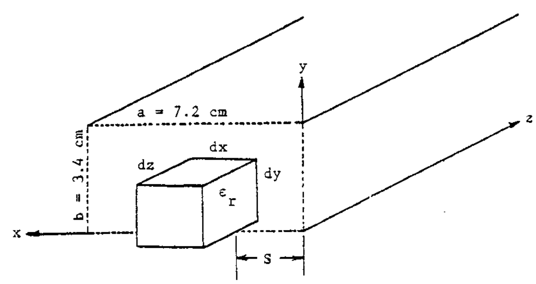


Figure 2. Comparison of convergence between direct truncation and partial summation.



CASE	ε_r	d_x	d_y	d_z	S
A	2.25 + j0.00045	4.9	1.4	3.05	1.15
B	4.25 + j0.0425	4.0	1.25	2.0	1.0
C	49.5 + j16.8	3.2	2.65	2.0	2.93

ε_r = relative dielectric constant at 2.65 GHz
d_x, d_y, d_z, S in cm.

Figure 3. Configurations of the three cases studied

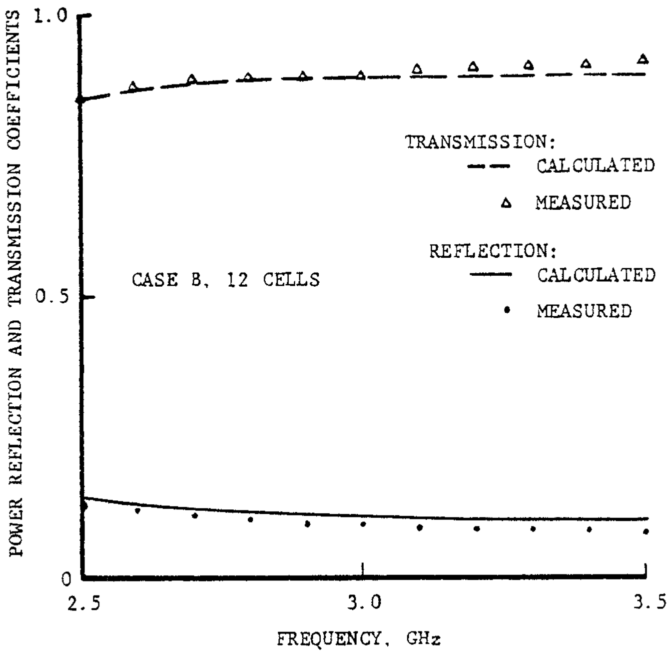


Figure 4. Comparison between calculated and measured reflection and transmission characteristics for case B of low dielectric constant.

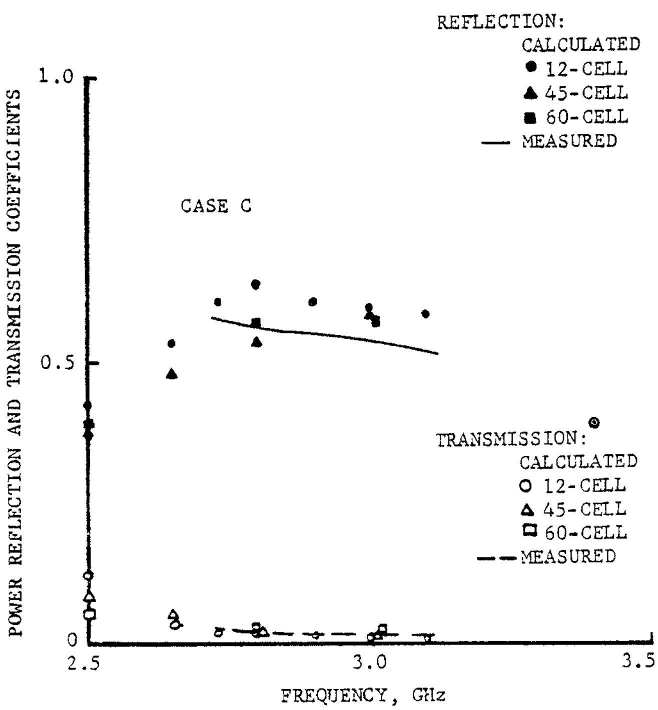


Figure 5. Comparison between measurements and calculations of various numbers of cells for case C of a high dielectric constant.

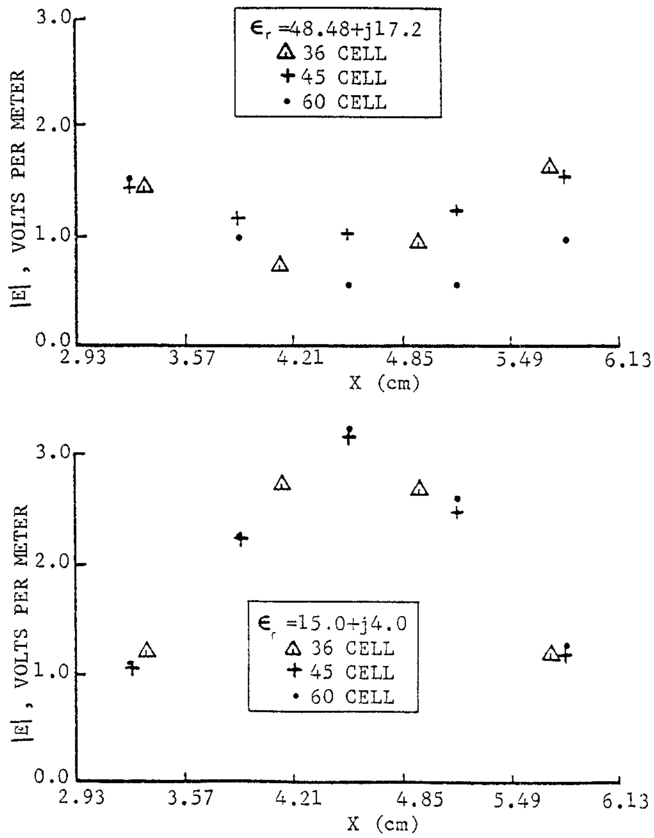


Figure 6. Convergence of field distribution (at y = 1.325 cm, z = 0.0) for case C at 2.8 GHz with high and medium dielectric constants.